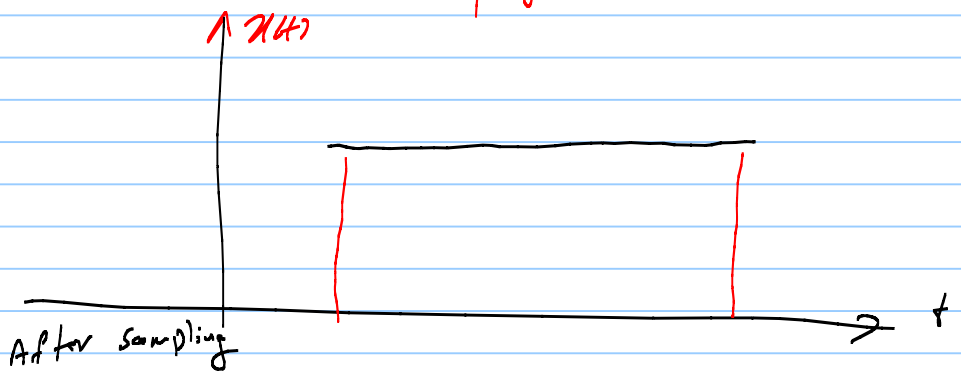
Note Title 4/11/2019



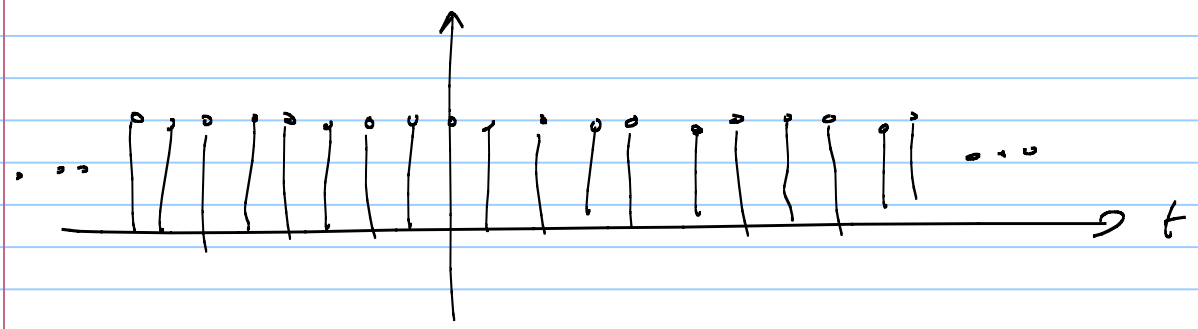


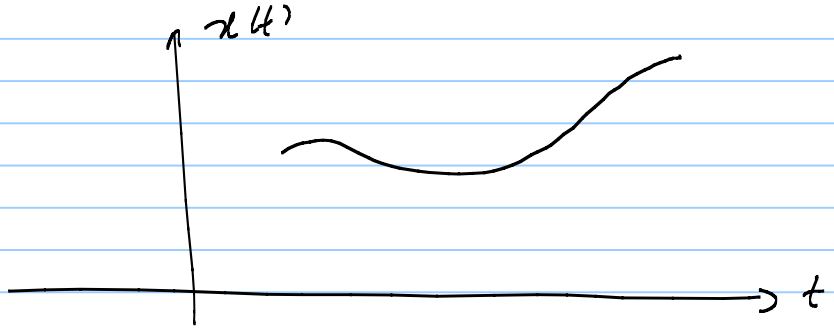
$$x[n] = x(t)\Big|_{t=nT} = x(nT) \Leftarrow$$

n = number of samples

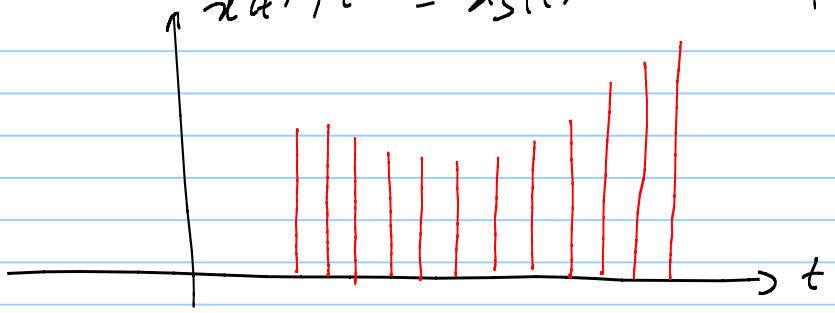
We can use the impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$





$x(t)\,p(t) = x_s(t)$ ← sampled signal



$$x_s(t) = x(t)\,p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

$\sum_{n=-\infty}^{\infty}\delta(t-nT)$ ← periodic function

$$p(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_s t}$$

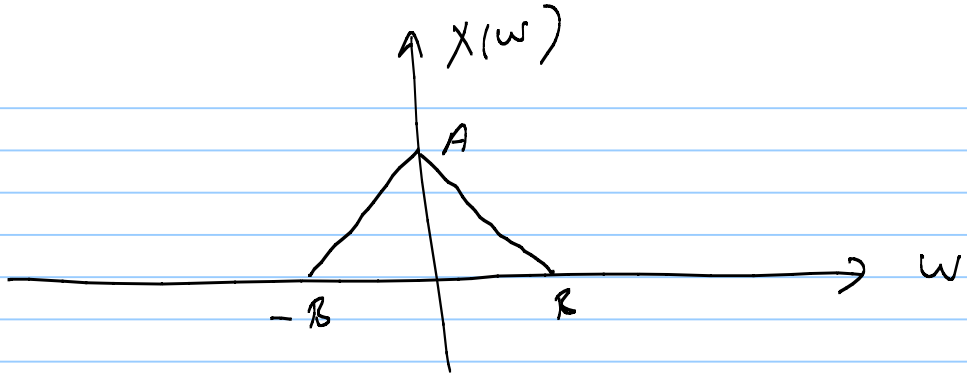
$$c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2}\delta(t-nT)\,e^{-jk\omega_s t}\,dt = \frac{1}{T}$$

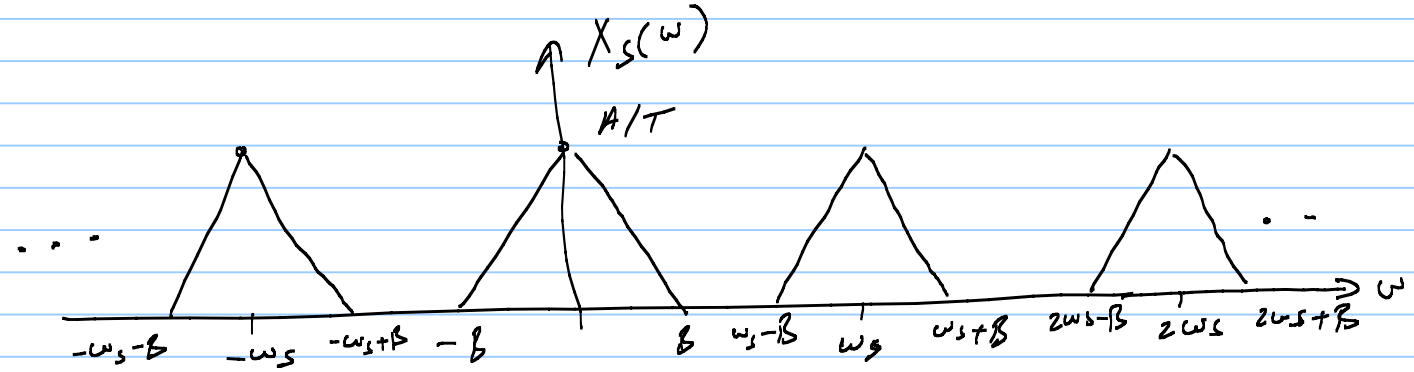
$$p(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

$$x_s(t) = x(t)\,p(t) = \frac{1}{T}x(t)\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

$$\therefore X_s(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

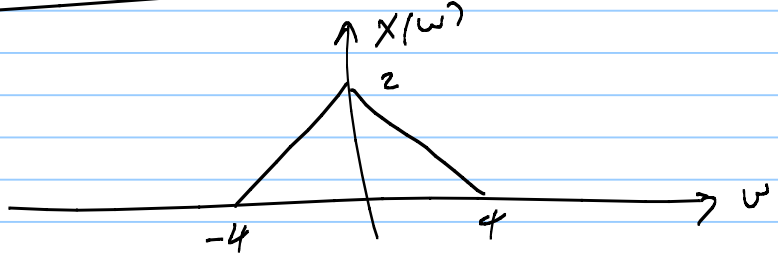
ω_s = Sampling Freq.





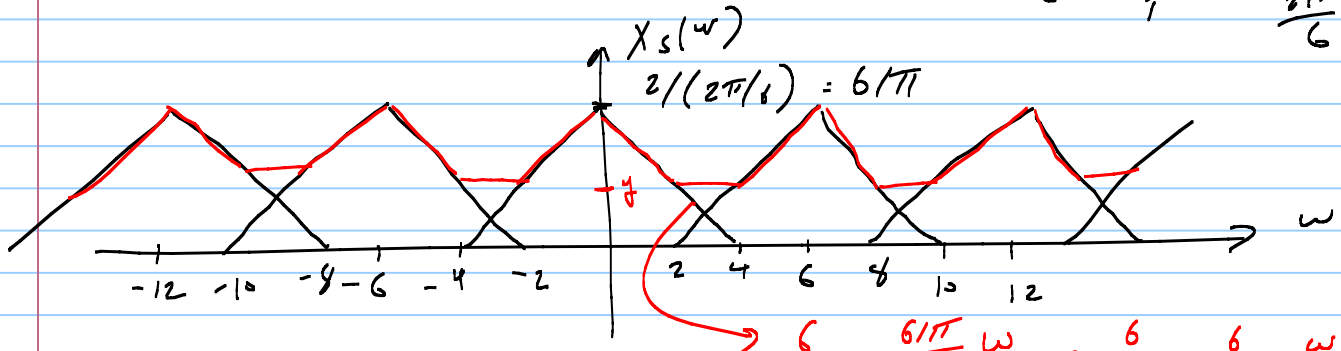
$\omega_s \geq 2B$

$$\therefore X_s(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$



$$T = \frac{2\pi}{6}$$

$$\omega_s = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi}{6}} = 6$$



$y = ?$

$$\frac{6}{\pi} - \frac{6/\pi}{4}\omega = \frac{6}{\pi} - \frac{6}{4\pi}\omega$$

$$y = \frac{6}{\pi} - \frac{6}{4\pi} \cdot (2) = \frac{6}{\pi} - \frac{12}{4\pi} = \frac{12}{4\pi} = \frac{3}{\pi}$$

Ex

$$H(\Omega) = 1 + e^{-j\Omega}$$

$$x[n] = 2 + 2\sin\left(\frac{\pi}{2}n\right)$$

$y[n] = ?$

$$|H(0)| = 1 + e^{0} = 2 \quad , \quad \angle H(0) = \angle 1 + e^{0} = \angle 2 = 0$$

$$|H(\tfrac{\pi}{2})| = |1 + e^{-j\frac{\pi}{2}}| = |1 + \cos(\tfrac{\pi}{2}) - j\sin(\tfrac{\pi}{2})|$$

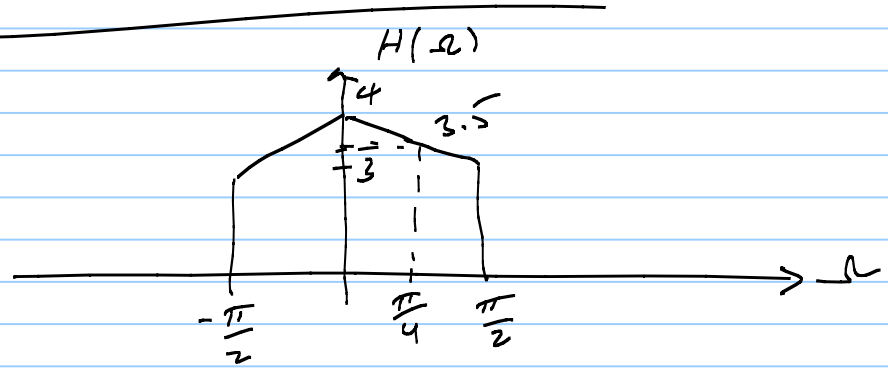
$$|1 - j| = \sqrt{2}$$

$$\angle H(\tfrac{\pi}{2}) = \angle 1 + e^{-j\frac{\pi}{2}} = \angle 1 - j = -45^{\circ} = -\frac{\pi}{4}$$

$$x[n] = 2 + 2\sin\left(\frac{\pi}{2}n\right)$$

$$y[n] = 4 + 2\sqrt{2}\sin\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)$$

Ex



$H(\Omega)$ is a real valued function

$$\angle H(\Omega) = 0$$

$$x[n] = 10 + 5\cos\left(\frac{\pi}{4}n\right) + 8\sin\left(\frac{3\pi}{2}n\right)$$

$y[n] = ?$

$$|H(0)| = 4$$

$|H(\frac{\pi}{4})| = 3.5$

$|H(\frac{3\pi}{4})| = 0$

$x[n] = 10 + 5\cos(\frac{\pi}{4}n) + 8\sin(\frac{3\pi}{2}n)$

$y[n] = 40 + 17.5\cos(\frac{\pi}{4}n) + 0$